Stochastic oscillations in circuit simulation

Thorsten Sickenberger and **Renate Winkler**

Institut für Mathematik, Humboldt-Universität zu Berlin, 10099 Berlin, Germany *

We consider the simulation of noisy electronic circuits with oscillatory solutions. For their transient noise simulation we use variable step-size two-step schemes for stochastic differential-algebraic equations. The performance of these methods in combination with a suitable step-size control strategy is illustrated by an industrial test application.

1 Transient noise analysis in circuit simulation

The scaling down from micrometer dimensions into nanometer range and the increasing operating frequency cause smaller signal-to-noise ratios. Noise has to be taken into account in time-domain simulations. For an implementation of an efficient transient noise analysis in an analog simulator, both an appropriate modelling and integration scheme is necessary.

We deal with the thermal noise of resistors as well as the shot noise of semiconductors that are modelled by additional sources of additive or multiplicative Gaussian white noise currents (see [1, 5]). The noise intensities are given by Nyquist's law and Schottky's formula. Combining Kirchhoff's Current law with the element characteristics and using the charge-oriented formulation yields a stochastic differential algebraic equation (SDAE) of the form

$$Aq(X(s))|_{t_0}^{t} + \int_{t_0}^{t} f(X(s),s)ds + \sum_{r=1}^{m} \int_{t_0}^{t} g_r(X(s),s)dW_r(s) = 0\ , \qquad t \in [t_0, T]\ , \tag{1}$$

where A is a constant singular matrix determined by the topology of the electrical network, the vector $q(x)$ consists of the charges and the fluxes, and x is the vector of unknowns consisting of the nodal potentials and the branch currents. The term $f(x,t)$ describes the impact of the static elements, $g_r(x,t)$ denotes the vector of noise intensities for the r-th noise source, and ξ is an m-dimensional vector of independent Gaussian white noise sources (see e.g. [1, 6]). One has to deal with a large number of equations as well as with a large number of comparatively small noise sources.

We understand (1) as a stochastic integral equation, where the second integral is an Itô-integral, and W denotes an m-dimensional Wiener process (or Brownian motion) given on the probability space $(\Omega, \mathcal{F}, P)$ with a filtration $(\mathcal{F}_t)_{t\geq t_0}$. The solution is a stochastic process depending on the time t and on the random sample ω. Typical paths are nowhere differentiable. Especially for oscillating solutions in circuit simulation one is interested in the phase noise. We therefore aim at the simulation of solution paths that reveal the phase noise and apply the concept of strong solutions and strong (mean-square) convergence of approximations.

2 Numerical Methods

In [4, 6] we present adaptations of known schemes for SDEs that are implicit in the deterministic part (drift) and explicit in the stochastic part (diffusion) of the SDAE. Designing the methods such that the iterates have to fulfill the constraints of the SDAE at the current time-point is the key idea to adapt known methods for the SDEs to (1). We consider stochastic analogues of the two-step backward differentiation formula (BDF$_2$) and the trapezoidal rule, where only the increments of the driving Wiener process are used to discretize the diffusion part (see [3, 4]).

Such schemes have only an asymptotic order of strong convergence of $1/2$. However, when the noise is small, the error behavior is much better. In fact, the errors are dominated by the deterministic terms as long as the step-size is large enough. In more detail, the error of both methods is bounded by $O(h^2 + \varepsilon h + \varepsilon^2 h^{1/2})$, when ε is used to measure the smallness of the noise $(g_r(x,t) = \varepsilon \hat{g}_r(x,t),\ r = 1,\ldots,m,\ \varepsilon \ll 1)$. Thus, we can expect order 2 behaviour if $h \gg \varepsilon$.

The smallness of the noise allows special estimates of the local error terms, which can be used to control the step-size. In [4] we present an error estimate and, based on this, a step-size control for stochastic linear multi-step methods with deterministic order 2 leading to adaptive step-size sequences that are uniform for all paths. There, the mean-square norm of the dominating local error term is estimated using values of the drift term and does not cost additional evaluations of the coefficients of the SDE or their derivatives.

* e-mail: sickenberger/winkler@math.hu-berlin.de

3 A voltage controlled oscillator

To be able to handle real-life problems, a slightly modified version of the schemes has been implemented in Qimonda's in-house analog circuit simulator TITAN, which has been used for the presented results.

As an industrial test application we us a voltage controlled oscillator that is a simplified version of a fully integrated 1.3 GHz VCO for GSM in 0.25 μm standard CMOS. The VCO is tunable from about 1.2 GHz up to 1.4 GHz. The unknowns of the VCO in the MNA system are the charges of the six capacities, the fluxes of the four inductors, the 15 nodal potentials and the currents through the voltage sources. This circuit contains 5 resistors and 6 MOSFETs, which induce 53 sources of thermal or shot noise. To make the differences between the solutions of the noisy and the noise-free model more visible, the noise intensities had been scaled by a factor of 500.

Numerical results obtained with a combination of the BDF_2 and the trapezoidal rule are shown in the left graph of Fig. 1, where we plotted the difference of the nodal potential $V(7) - V(8)$ of node 7 and 8 versus time. The solution of the noise-free system is given by a dashed line. Four sample paths (dark solid lines) are shown. They cannot be considered as small perturbations of the deterministic solution, phase noise is highly visible.

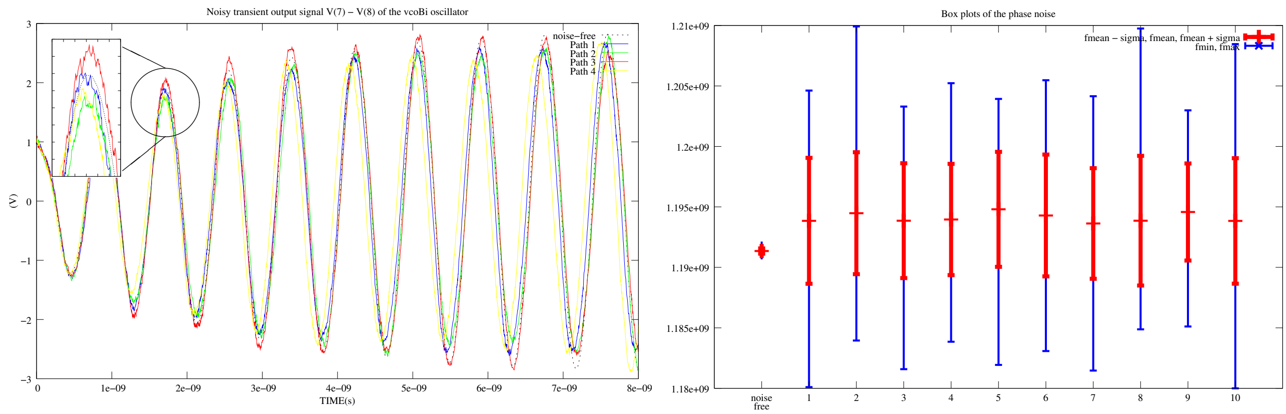

Fig. 1 Noisy transient output signal of a VCO (left) and boxplots of the phase noise (right), scaled by a factor of 500.

To analyze the phase noise we performed 10 simultaneous simulations with different initializations of the pseudo-random numbers. In a postprocessing step we computed the length of the first 50 periods for each solution path and then from these the corresponding frequencies. In the right graph of Fig. 1 the mean μ of the frequencies (horizontal lines), the smallest and the largest frequencies (boundaries of the vertical thin lines) and the boundaries of the confidence interval $\mu \pm \sigma$ (the plump lines) are presented, where σ was computed as the empirical estimate of the standard deviation. The mean appears increased and differs by about $+0.25\%$ from the noiseless, deterministic solution. Further on, the frequencies vacillate from 1.18 GHz (-0.95%) up to 1.21 GHz ($+1.55\%$). So the transient noise analysis shows that the voltage controlled oscillator runs in a noisy environment with increased frequencies and smaller phases, respectively.

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